Question 2 **(4 marks)**

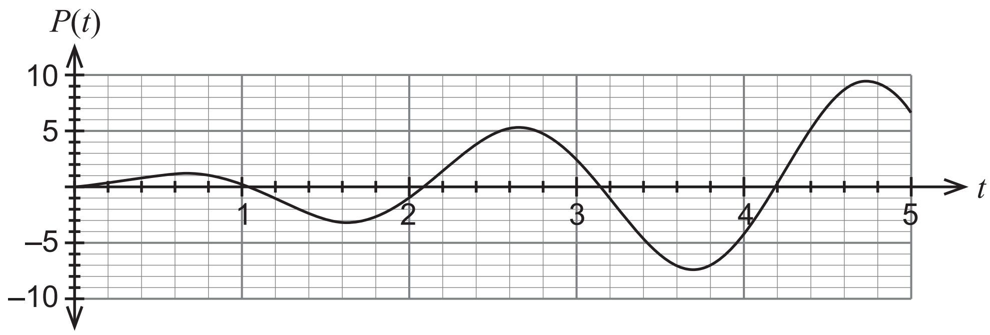
If $h(x) = \dfrac{e^{-x}}{\cos x}$, then evaluate $h'(\pi)$.

Question 7 **(9 marks)**

A company's profit, in millions of dollars, over a five-year period can be modelled by the function:

$P(t) = 2t\sin(3t) \quad 0 \le t \le 5$ where t is measured in years.

The graph of $P(t)$ is shown below.



(a) Differentiate $P(t)$ to determine the marginal profit function, $P'(t)$. (2 marks)

(b) Calculate the rate of change of the marginal profit function when $t = \frac{\pi}{18}$ years. (4 marks)

(c) Use the increments formula at $t = \frac{7\pi}{6}$ to estimate the change in profit for a one month change in time. (3 marks)

End of section

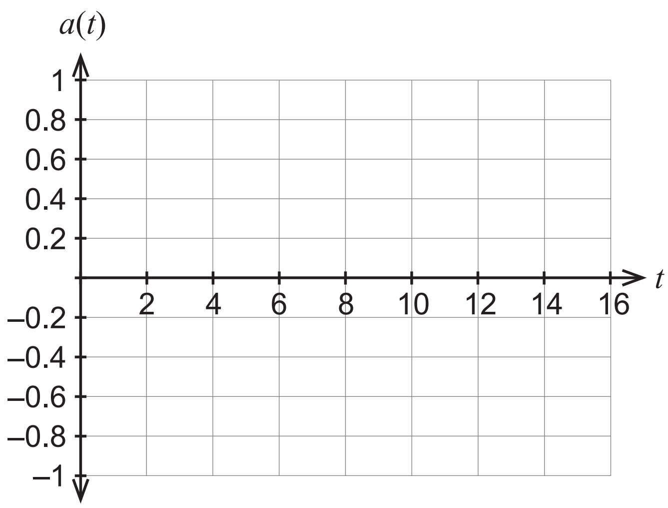
Question 9 **(8 marks)**

It takes an elevator 16 seconds to ascend from the ground floor of a building to the sixth floor. The velocity of the elevator during its ascent is given by

$$v(t) = \frac{9\pi}{16}\sin\left(\frac{\pi t}{16}\right) \text{ m/s.}$$

The velocity, v, is measured in metres per second, while the time, t, is measured in seconds.

(a) Determine the acceleration of the elevator during its ascent and provide a sketch of the acceleration function for $0 \le t \le 16$. (2 marks)



(b) With reference to your answer from part (a), explain what is happening to the velocity of the elevator in the interval $0 < t < 8$ and in the interval $8 < t < 16$. (3 marks)

(c) Suppose that the ground floor has displacement $x = 0$ m. Determine the displacement function of the elevator and hence determine the height above the ground floor of the sixth floor. (3 marks)

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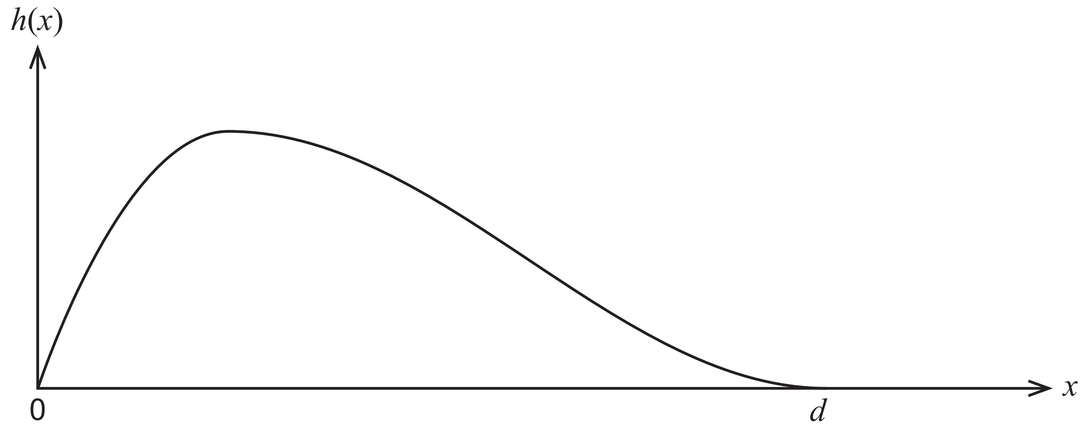
Question 15 **(14 marks)**

A wall in a new Western Australian hotel is to feature a rolling, wave-shaped window. Engineers have modelled the top edge of the wave shape by joining together two functions,

$$h_1(x) = 4 - 4(x-1)^2, \quad 0 \le x \le 1 \text{ and}$$

$$h_2(x) = a(\cos(x-1) + 1), \quad 1 < x \le d \quad a, d \text{ constants.}$$

The functions give the height, h, above ground level of the top edge of the window measured in metres. The origin is defined as the leftmost point of the window which is at ground level and x is the horizontal distance to the right of the origin measured in metres. The graph of the two functions is shown below.

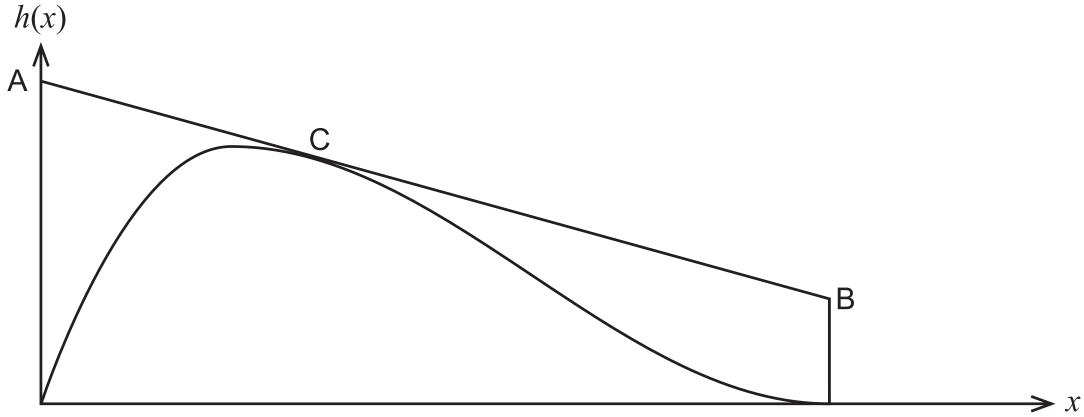


(a) Determine the value of the constant a in the function $h_2(x) = a(\cos(x-1) + 1)$. (3 marks)

(b) Determine the length of the bottom edge of the window. (2 marks)

(c) Determine the volume of glass required for the window if it has a uniform thickness of 3 cm. (5 marks)

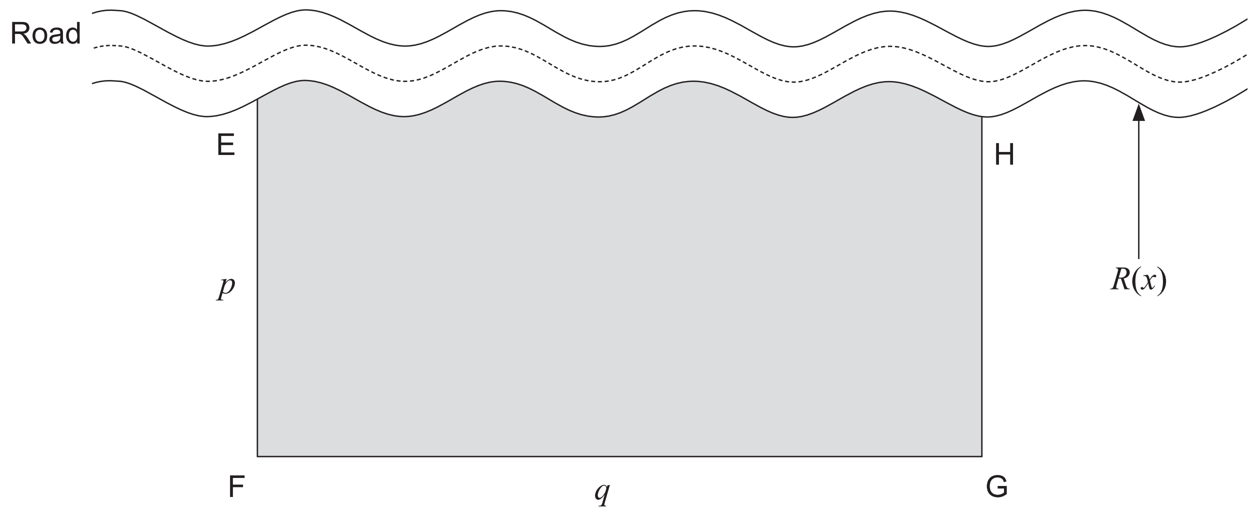
The top edge of the wall, shown as the line AB below, is to just touch the window at the point C shown below. Point A is 1.39 m above the point B.



(d) How high is point C above the ground? (4 marks)

Question 17 **(12 marks)**

David and Katrina have a small farm and wish to fence off an area of their land so they can raise sheep. The area they have chosen has one border along a road as shown in the diagram below.



The enclosure is shown as the shaded area above and has right angles at points F and G. David and Katrina want the combined lengths of the fencing from E to F and F to G to equal 500 metres. Let the length of fence EF be equal to p metres and the length of fence FG be equal to q metres. If we locate the origin at point F and the x-axis along the line FG, the equation defining the fence along the road is given by:

$$R(x) = 10\sin\left(\frac{x}{15}\right) + p$$

(a) Show that the equation defining the area of the enclosure, $A(q)$, can be given in terms of q as follows:

$$A(q) = 500q - 150\cos\left(\frac{q}{15}\right) - q^2 + 150$$

(4 marks)

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(b) Determine, to the nearest metre, the value of q that will allow the sheep to graze over the maximum area and state this maximum area. (4 marks)

The length of the fence from E to H is given by the equation:

$L_{EH} = \int_0^q \sqrt{1 + (R'(x))^2}\, dx$, where $R'(x)$ is the first derivative of $R(x)$.

(c) (i) Determine $R'(x)$. (1 mark)

(ii) Hence determine the total length of fencing required by David and Katrina to enclose their sheep with maximum area for grazing. (3 marks)

End of questions